

1 Probability

1.1 The typewriting monkey

Okay, so it's only a monkey to make it 'biological'. It could be a typewriting anything (except most things don't type). But the point is, whatever it is (let's call it a monkey) is sitting at a keyboard, and it presses the keys pretty much randomly. The probability that it hits a letter key is about $2/3$ and this is a much reduced keyboard, so the only other keys are number keys. For some reason, we watch the monkey whilst it presses ten keys. (i) What's the probability that it pressed only number keys? (ii) What's the probability that it pressed fewer than three number keys? (iii) What's the probability that it alternated number and letter keys?

Now, let's assume we find a different monkey with completely different properties. For this monkey, after it presses a number key, the probability that the next key press is also a number is $3/5$; if it presses a letter key, the next key press is also a letter with probability $4/5$. This monkey has been sitting in a room all day, pressing these keys. At about 4 in the afternoon, we come in to the room, and watch the monkey whilst it presses 3 keys. (iv) what's the probability that it starts by pressing a letter key? (v) What's the probability that it presses three letter keys? (vi) What's the probability that it presses two of one type of key and one of the other type? (vi) If I tell you that the third key press was a letter, what (on a balance of probability) was the first letter?

1.2 Some tedious algebra

So I'm not even going to pretend this is biological. I just want you to do a bit of maths. I'd like you to find alternative expressions (but not too alternative...) for: (i) $(x + 1)^4$, (ii) $\exp(a)\exp(b)$ (iii) $\frac{1}{(x+1)(x-1)}$ (iv) x^{-7} (v) $\sqrt{x + 1}$ (vi) $-(-7)$ (vii) $\frac{x+1}{x^2-1}$

1.3 More monkey business

Back to that monkey, though... it goes for a walk in the evening, setting off just after watching Neighbours (which ends at 6.00). We plot the distance it gets away from the monkey house against time. It's the sort of thing you get to do when you're doing a PhD. They even let you do this sort of thing in your third year project, sometimes. Anyway, I reckon that the distance the monkey gets from the house in miles is given approximately by the function $t(t^2 - 4t + 4)$, where t is time (i) When does the monkey arrive back home? (ii) What does it do then? (iii) On its little walk, how far did it get from the house? (iv) When did that happen? (v) Comment on the validity of the function $t(t^2 - 4t + 4)$ for negative t , and draw a graph of the monkey's path.

1.4 Monkey puzzle trees

Everyone knows, of course, that monkeys don't interact well with monkey puzzle trees. So when the monkey finds one (10 miles from home), it eats it. Its rate of consumption of monkey puzzle tree leaves is $2t^2$ leaves

per minute, where t is the time after the monkey started eating the tree leaves, in minutes. When the monkey finishes eating the leaves, it spends half an hour uprooting the tree. It then tosses the stump in the air wanders off. Given that gravity causes an acceleration of $10ms^{-2}$ the tree stump manages a maximum height of 20 metres. It takes the monkey about 5 minutes to eat up all of the leaves on the tree. (i) How far is the monkey puzzle tree from home? (ii) How many leaves were on the tree? (iii) What's the fastest rate of consumption that the monkey manages? (iv) At what time (precisely) does the tree stump land back on the ground?

1.5 Monkeying around

Had the weather not taken a turn for the worse, that would have been the end of my monkey. But this monkey hates the rain, and with each hour that passed, the skies grew darker... but that sort of thing always happens at night. However, each day, there's a probability of about $1/5$ of it raining, and as soon as that happens, the monkey will head home. By this time, the monkey will be hanging out (as it were) in the forest just 30 miles from home, and it will take it a whole 5 days to get back home again. If the weather holds off for 10 days, the monkey will be so much in need of another spell using its typewriter that it will come home anyway. What's the mean time until the monkey comes home?

1.6 Monkey nuts

Identify a reason or two why this whole thing is a bit crazy. Mathematical reasons are preferred, of course... Did you have to make any additional assumptions to answer the questions?