

EMB test, January 2003

Answer all questions. The duration of this test is 60 minutes. You are expected to spend approximately 15 minutes on each question

Question 1

1. Solve:

$$\begin{aligned}x + y + z &= 9 \\x &= 2y - 2 \\x + y + 1 &= z\end{aligned}$$

2. Solve:

$$3x^2 + x = 52$$

3. Solve:

$$\frac{x}{2x+1} = 7$$

4. Solve:

$$\begin{aligned}x + y &= 7 \\x^2 + y^2 &= 25\end{aligned}$$

5. What is 2 radians expressed as degrees?

Question 2

1. Differentiate the following expression for y with respect to x ,

$$y = 4x^2 + \frac{7}{x}$$

2. Differentiate the following expression for y with respect to x ,

$$y = 4 \sin(7 \exp(x))$$

3. Differentiate the following expression for y with respect to x ,

$$y = x^2 \sin(2x) + 9$$

4. Differentiate the following expression for y with respect to x ,

$$y = (x^2 + \sin(x))^3$$

5. Find the following integral:

$$\int 2 \sin(3x) dx$$

6. Evaluate the following integral, and draw a sketch to illustrate the area it represents:

$$\int_1^3 x^2 dx$$

Question 3

A man, sitting on the ground, is attempting to measure the height of a tree. The base of the tree is 50m away, and the ground is horizontal. The tree is not quite vertical and leans slightly towards him, making an angle of 80° with the ground. The tree subtends an angle of 10° at the man's eyes, with respect to the horizontal. What is the height of the top of the tree (that is, how many meters is the top of the tree above the ground)? How tall is the tree? You can assume the man's height is negligible in comparison with the height of the tree. The man returns to the same spot a year later to discover that the tree now subtends an angle of 12° at the man's eyes, with respect to the horizontal. The tree is still leaning to the same extent. What has been the tree's average rate of growth per day in the course of the year?

Question 4

A patient takes a drug, by mouth, and the concentration of the drug in the bloodstream is then given by the equation $c = 5te^{-0.2t}$ where c is concentration in $\mu g/l$ and t is time in hours after the drug was taken.

1. When will the drug be most concentrated in the blood stream? What is the maximum concentration?
2. Sketch the resulting variation in concentration over time.
3. It is necessary to give the patient the maximum dose of the drug possible. However, concentrations in the blood stream higher than $10\mu g/l$ are fatal. A dose of A milligrams taken by mouth causes the concentration of the drug in the bloodstream to follow the curve $c = Ate^{-0.2t}$ where c is concentration in $\mu g/l$ and t is time in hours after the drug was taken. What dose should be given?