

EMB exam, January 2001

Answer all questions. The duration of this exam is 60 minutes. You are expected to spend approximately 15 minutes on each question

Question 1

1. Solve:

$$x + y + z = 9 \quad (1)$$

$$x = 2y - 2 \quad (2)$$

$$x + y + 1 = z \quad (3)$$

2. Solve:

$$3x^2 + x = 52 \quad (4)$$

3. Sketch:

$$x = \frac{y}{2y - 1} \quad (5)$$

4. If $y = at^2$, where a is a constant, show that a graph of $\ln y$ against $\ln t$ would be a straight line.

What would the gradient be, and where would it cross the axes?

Question 2

A man, sitting on the ground, is attempting to measure the height of a tree. The base of the tree is 50m away, and the ground is horizontal. The tree is not quite vertical and leans slightly towards him, making an angle of 80° with the ground. The tree subtends an angle of 10° at the man's eyes, with respect to the horizontal. How high is the tree, and how high would it be if it stood up straight? You can assume the man's height is negligible in comparison with the height of the tree.

Question 3

A population of rabbits, size n , is growing according to the equation, $\frac{dn}{dt} = 2n$, where t is time in years.

1. Show that $n = 3e^{2t}$ is a solution of this equation. Show that $n = 7e^{2t}$ is also a solution. Are there any other solutions?
2. Biologically, what does the 2 signify?
3. Sketch the solution $n = 3e^{2t}$ starting at $t = 0$ and ending at $t = 5$. Why is the model unrealistic as t gets large?
4. How long does it take the rabbit population to double in size?

Question 4

A patient takes a drug, by mouth, and the concentration of the drug in the bloodstream is then given by the equation $c = 5te^{-0.2t}$ where c is concentration in $\mu g/l$ and t is time in hours after the drug was taken.

1. When will the drug be most concentrated in the blood stream? What is the maximum concentration?
2. A second dose is given, 5 hours after the first. Give an equation for the total concentration in the blood as a result of the two doses, assuming that the effects of taking two doses are simply additive. What is the maximum concentration now?
3. Sketch the resulting variation in concentration over time?